ÉRETTSÉGI VIZSGA • 2019. május 7.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ
ÍRÁSBELI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
    - correct calculation: *checkmark*
    - principal error: *double underline*
    - calculation error or other, not principal, error: *single underline*
    - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
    - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
    - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1. a)		
$4\cdot 2^{2x}+31\cdot 2^{x}-8=0$	1 point	
This equation is quadratic for 2^x.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$2^x=-8$ or $2^x=\frac{1}{4}$	1 point	
The first case is not possible (as $2^x>0$ for all real values of x).	1 point	
(Because of the strict monotonicity) the second case gives $x=-2$.	1 point	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**6 points**	

1. b)		
$\sin x(4\sin^2 x-1)=0$	1 point	
$\sin x=0$ or $\sin^2 x=\frac{1}{4}$	1 point	
If $\sin x=0$, then $x=k\pi$, $k\in \mathbf{Z}$.	1 point	
If $\sin^2 x=\frac{1}{4}$, then $\sin x=\frac{1}{2}$ or $\sin x=-\frac{1}{2}$.	1 point	$\lvert\sin x\rvert=\frac{1}{2}$,
In the first case: $x=\frac{\pi}{6}+k\cdot 2\pi$ or $x=\frac{5\pi}{6}+k\cdot 2\pi$,	1 point	*therefore* $x=\frac{\pi}{6}+k\pi$,
in the second case $x=\frac{7\pi}{6}+k\cdot 2\pi$ or $x=\frac{11\pi}{6}+k\cdot 2\pi$, $k\in \mathbf{Z}$.	1 point	*or* $x=-\frac{\pi}{6}+k\pi$, $k\in \mathbf{Z}$.
Check by substitution or reference to equivalent steps.	1 point	
Total:	**7 points**	

Notes:
1. Deduce a total of 1 point if the candidate includes the periods in their answers, but does not give the condition $k\in \mathbf{Z}$.
2. Award a maximum of 5 points if the candidate gives their solutions without the appropriate periods.
3. Award a maximum of 6 points if the candidate gives their answer in degrees.
4. Award a maximum of 4 points if the candidate gives their answer in degrees, without the appropriate periods.

2. a)		
Let n be the number of towns ($n \geq 2$). The number of edges in a complete graph of n vertices is $\frac{n(n-1)}{2}$,	1 point	
The number of edges in a tree graph of n vertices is $n-1$.	1 point	
The equation is $\frac{1}{3}\cdot\frac{n(n-1)}{2}=n-1$.	1 point	$n^2-7n+6=0$,
As, according to the text of the question, $n-1\neq 0$, the equation may be divided by it.	1 point	*The two roots are* 6 *and* 1 *but, according to the text,* $n\neq 1$.
The number of towns is therefore $n = 6$.	1 point	
Check: A complete graph of 6 vertices has 15 edges, while a tree graph of 6 vertices has 5 edges. So, two thirds of the 15 edges must, in fact, be deleted.	1 point	
Total:	**6 points**	

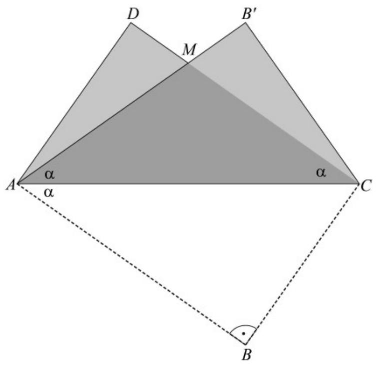
2. b) Solution 1		
A total of $\frac{10\cdot 9}{2}=45$ games were played.	1 point	
(Each draw increases the total score by 2, each game that is not a draw increases it by 3.) Assuming the number of draws is x, the number of games that a team won is $45-x$. According to the text of the question: $x\cdot 2+(45-x)\cdot 3=130$.	2 points	*draw: x games* *not draw: y games* $x+y=45$ *and* $2x+3y=130$
The solution is $x = 5$ (this is the number of draws).	1 point	*The solution of the equation system is:* $(x; y) = (5; 40)$.
Check: The 5 draws added ($5\cdot 2=$) 10 points to the total, while the other 40 games added ($40\cdot 3=$) 120 points. The total for all teams is, in fact, 130 points.	1 point	
Total:	**5 points**	

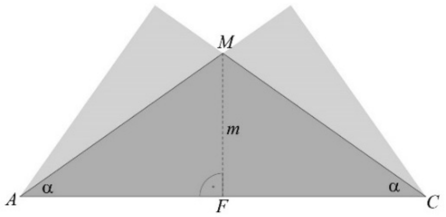
2. b) Solution 2		
A total of $\frac{10\cdot 9}{2}=45$ games were played.	1 point	
(Each draw increases the total score by 2, each game that is not a draw increases it by 3.) Should all games have ended with one of the teams winning, the total score would have been $45\cdot 3=135$ points.	2 points	
However, the total score is 1 point less per draw,	1 point	
and so ($135-130=$) 5 games must have been draws.	1 point	
Total:	**5 points**	

3. a)		
There are 7! = 5040 appropriate seven-digit numbers.	1 point	
If each of those numbers is written on a 0.5 cm × 2 cm slip of paper, the total area will be $5040 \cdot 0.5 \cdot 2 = 5040$ (cm^2).	1 point	
The combined area of eight A4 sheets is $8 \cdot 21 \cdot 29.7 = 4989.6$ (cm^2),	1 point	
which means eight A4 sheets will not be enough.	1 point	
Total:	**4 points**	

Note: Award a maximum of 3 points if the candidate proves that 8 sheets will not be enough by cutting an A4 sheet into 0.5 cm × 2 cm *rectangles in a particular manner, but does not prove that there is no way to cut the same sheet into more rectangles in some other way.*

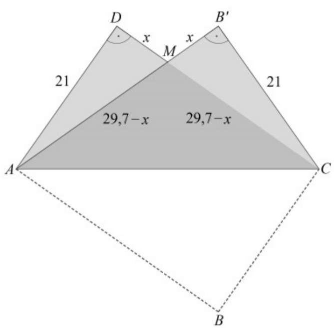
3. b)		
There are 6! = 720 seven-digit numbers among the above that begin with the digit 1.	1 point	*As there are 7 digits (none of them* 0*), one seventh of all appropriate numbers will begin with the digit* 1*, that is* 7! : 7 = 720 *numbers.*
721 = 6! + 1,	1 point	
(As the numbers are arranged in increasing order) the 721st number will be the smallest among those beginning with the digit 2.	1 point	
Therefore, it will really be 2 134 567.	1 point	
Total:	**4 points**	

3. c) **Solution 1**		
 Using the symbols of the diagram: all angles α (*BAC, B'AC* and *DCA*) are equal which makes the triangle *AMC* an isosceles one.	1 point	
(Apply the Pythagorean Theorem) in the right triangle *ABC*: $AC = \sqrt{29.7^2 + 21^2} \approx 36.4$ (cm),	1 point	
and also $\tan\alpha = \frac{21}{29.7}$ ($\alpha \approx 35.3°$).	1 point*	

 the height that belongs to the base AC of triangle AMC is: $m = \frac{AC}{2} \cdot \tan\alpha$ (≈ 12.9 cm),	1 point*	
The area of the overlap is therefore $\frac{AC \cdot m}{2} \left(= \frac{AC^2}{4} \cdot \tan\alpha \right) \approx 234\ \text{cm}^2$.	1 point*	*Calculating with rounded values, the area will be approximately* 235 *cm*^{2}.
Total:	**5 points**	

*Note: the points marked by * may also be given for the following reasoning:*

$\tan\alpha = \frac{21}{29.7}$,	1 point	
$\alpha \approx 35.26°$ and $180° - 2\alpha \approx 109.5°$,	1 point	
The area of the overlap is therefore $A_{AMC} = \frac{AC^2 \sin^2\alpha}{2\sin(180° - 2\alpha)} \approx 234\ \text{cm}^2$.	1 point	

3. c) Solution 2		
 Using the symbols of the diagram: if $DM = x$, then (due to the line symmetry) $AM = MC = 29.7 - x$.	1 point	
(Apply the Pythagorean Theorem) in the right triangle AMD: $21^2 + x^2 = (29.7 - x)^2$.	1 point	
$441 = 882.09 - 59.4x$ $x \approx 7.426$ (cm),	1 point	
$A_{AMD} = \frac{21 \cdot 7.426}{2} \approx 77.97$ (cm^2)	1 point	
(The area of the overlap is equal to the difference of the areas of triangles ACD and AMD.) $A_{ACM} = \frac{29.7 \cdot 21}{2} - A_{AMD} \approx 234\ \text{cm}^2$.	1 point	
Total:	**5 points**	

4. a) **Solution 1**		
One direction vector of the line of segment AB is $\mathbf{v}(10; 5)$, a possible normal vector is $\mathbf{n}(1; -2)$.	1 point	*The equation of line AC is* $53x - 96y = 1200$, *the equation of line BC is* $43x - 76y = 1000$.
The equation of line AB is $x - 2y = 25$.	1 point	
(Plugging the coordinates of point C into this equation:) $48 - 2 \cdot 14 \neq 25$, which means the three points are not collinear.	2 points	*Point B is not on the line AC as* $1250 \neq 1200$. *Point A is not on the line BC as* $950 \neq 1000$.
Total:	**4 points**	

4. a) **Solution 2**		
$\overrightarrow{AB} = (10; 5)$ and $\overrightarrow{BC} = (38; 21.5)$	1 point	$\overrightarrow{AC} = (48; 26.5)$
(The starting end endpoints of two adjoining vectors are collinear if and only if the vectors are parallel, i.e. the ratios of their first and second coordinates are equal.) The ratio of the coordinates of vector $\overrightarrow{AB}$ is $2:1$, while the ratio of the coordinates of vector $\overrightarrow{BC}$ is different.	2 points*	
The two vectors are not parallel, and so the three points may not be collinear either.	1 point	
Total:	**4 points**	

*Note: the 2 points marked by * may also be given for the following reasoning:*
(If two vectors have the same starting point, this and the two endpoints may only be collinear if the vectors are parallel, i.e. the ratios of their first and second coordinates are equal.) The ratio of the coordinates of vector $\overrightarrow{AB}$ *is* $2:1$, *while the ratio of the coordinates of vector* $\overrightarrow{AC}$ *is different.*

4. a) **Solution 3**		
$AB = \sqrt{100+25} = \sqrt{125}$ (≈ 11.180) $AC = \sqrt{2304+702.25} = \sqrt{3006.25}$ (≈ 54.829) $BC = \sqrt{1444+462.25} = \sqrt{1906.25}$ (≈ 43.661)	2 points	
As $AB + BC > AC$, the triangle ABC exists. The points A, B, C are, therefore, not collinear.	2 points	
Total:	**4 points**	

4. a) Solution 4		
$\overrightarrow{BA}=(-10;-5)$ and $\overrightarrow{BC}=(38;21.5)$	1 point	
$BA=\sqrt{100+25}=\sqrt{125}$ (≈ 11.180) $BC=\sqrt{1444+462.25}=\sqrt{1906.25}$ (≈ 43.661)	1 point	
The scalar product of vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ is calculated both ways: $(-10)\cdot 38+(-5)\cdot 21.5=\sqrt{125}\cdot\sqrt{1906.25}\cdot\cos(ABC\angle)$	1 point	
$ABC\angle \approx 177.1°$. The points A, B, C are, therefore, not collinear.	1 point	$BAC\angle \approx 2.3°$ $ACB\angle \approx 0.6°$
Total:	**4 points**	

4. b)		
(The set of points in the plane equidistant from points A and B is the perpendicular bisector of segment AB.) The midpoint of segment AB is $F(5;-10)$; one normal vector of the perpendicular bisector is $\mathbf{n}(2;1)$,	1 point	
the equation is $2x+y=0$.	1 point	
(The set of point in the plane that are 1000 metres (i.e. 50 units) from point C is a circle with centre C and radius 50 units.) The equation of the circle is $(x-48)^2+(y-14)^2=50^2$.	1 point	
(Possible positions for point D are the points of intersection of the perpendicular bisector and the circle:) the equation system $\left.\begin{array}{r}-2x=y\\(x-48)^2+(y-14)^2=50^2\end{array}\right\}$ has to be solved.	1 point	
Express y from the first equation and substitute it into the second: $(x-48)^2+(-2x-14)^2=50^2$,	1 point	$\left(-\frac{y}{2}-48\right)^2+(y-14)^2=50^2$
which leads to the equation: $5x^2-40x=0$.	1 point	$\frac{5}{4}y^2+20y=0$
Here $x_1=0$ and $x_2=8$,	1 point	
$y_1=0$ and $y_2=-16$ (the possible positions of point D are (0; 0) and (8; –16)).	1 point	
The possible values (in metres) of the distance AD are $20\cdot 12.5=250$ or $20\cdot\sqrt{(0-8)^2+((-12.5)-(-16))^2}$ ($\approx 20\cdot 8.73$) ≈ 175.	2 points	*Deduce a total 1 point if the candidate does not give the real distances.*
Total:	**10 points**	

II.

5. a) **Solution 1**		
Let a be the result of the first roll (the first term of the sequence) and let d be the result of the second roll (the common difference of the sequence). The sum of the first 10 terms of the sequence is S_{10} $= \frac{(2a+9d)\cdot 10}{2} = 10a + 45d$.	2 points	
Now $10a + 45d < 100$, i.e. $2a + 9d < 20$ (here $a, d \in \{1; 2; 3; 4; 5; 6\}$).	1 point	
If $d \geq 2$ then the inequality does not have a solution.	1 point	
If $d = 1$ then the inequality will be true for $a \in \{1; 2; 3; 4; 5\}$, so there are 5 suitable sequences.	1 point	
Total:	**5 points**	

5. a) **Solution 2**		
If the second roll is a 1 and the first roll is 1, 2, 3, 4 or 5, then the sum of the first ten terms will be 55, 65, 75, 85 or 95, respectively. All these are suitable solutions.	2 points	
If the second roll is a 1 and the first is a 6, then the sum of the first ten terms is 105, not a suitable solution.	1 point	
If the second roll is 2 or more, then the sum of the first ten terms is at least $1 + 3 + 5 + \ldots + 19 = 100$, and so none of these will be a solution.	1 point	
There are five possible sequences.	1 point	
Total:	**5 points**	

5. b)		
If all four digits are the same, then it is an arithmetic sequence. This gives 9 different possibilities.	1 point	
Consecutive terms of an arithmetic sequence will also be obtained if the four digits (in any order) are 1, 2, 3, 4; 4, 5, 6, 7; 2, 3, 4, 5; 5, 6, 7, 8; 3, 4, 5, 6; 6, 7, 8, 9;	1 point	
1, 3, 5, 7; 2, 4, 6, 8; 3, 5, 7, 9. (The absolute value of the common difference of the sequence may not be greater than 2.)	1 point	
Four different digits determine $4! = 24$ different numbers, so the 9 cases listed above will give $(9 \cdot 24 =)$ 216 different four-digit numbers.	1 point	
The total number of appropriate four-digit numbers is $9 + 216 = 225$.	1 point	
Total:	**5 points**	

5. c) **Solution 1**		
If Janka's fifth roll is a 3, yielding a mean of 3, the sum of the results of the five rolls will be 15. The sum of the first four rolls is therefore 12.	1 point	
If the fifth roll is a 5, yielding a single mode of 5, there must have been at least one other 5 among the first four results, too.	1 point	
In this case (as the sum of the other three results is 7) the first four rolls must have been 1, 1, 5, 5 or 1, 2, 4, 5 or 1, 3, 3, 5 or 2, 2, 3, 5 (in any particular order).	1 point	
If the fifth roll is a 4 and the median is also a 4, then the options 1, 3, 3, 5 and 2, 2, 3, 5 listed above will be wrong.	1 point	*The last two options are wrong as the single mode is not 5.*
The other two options both satisfy the condition about the mode, too.	1 point	*The other two options both satisfy the condition about the median, too.*
The results of the first four rolls must have been 1, 1, 5, 5 or 1, 2, 4, 5 (in any particular order).	1 point	
Total:	**6 points**	

5. c) **Solution 2**		
If Janka's fifth roll is a 3, yielding a mean of 3, the sum of the results of the five rolls will be 15. The sum of the first four rolls is therefore 12.	1 point	
Let the results of the first four rolls be *a, b, c, d*, listed in a non-decreasing order. According to the condition about the median, c may only be a 4 or a 5 (as $c \leq 3$ is not allowed). (In case $c = d = 6$, the sum $a + b = 0$ which is not possible.)	1 point	
According to the condition about the mode only $d = 5$ would be correct (in case $d = 6$, $c = 5$ and $a + b = 1$ which is not possible).	1 point	
If $c = 4$ then (as $a + b + c = 7$) $a = 1$, $b = 2$ and so the first four rolls must have been 1, 2, 4, 5 (in any particular order).	1 point	
If $c = 5$, then (as $a + b + c = 7$) $a = 1$, $b = 1$ and so the first four rolls must have been 1, 1, 5, 5 (in any particular order).	1 point	
Both possible solution also satisfy the conditions about the median and the mode, too.	1 point	
Total:	**6 points**	

5. c) Solution 3		
If Janka's fifth roll is a 3, yielding a mean of 3, the sum of the results of the five rolls will be 15. The sum of the first four rolls is therefore 12.	1 point	
If the sum of the first four rolls is 12, then these must have been (listed in non-decreasing order): 1, 1, 4, 6; 2, 2, 2, 6; 1, 1, 5, 5; 2, 2, 3, 5; 1, 2, 3, 6; 2, 2, 4, 4; 1, 2, 4, 5; 2, 3, 3, 4; 1, 3, 3, 5; 3, 3, 3, 3. 1, 3, 4, 4;	2 points	
The condition about the median forbids the following cases: 1, 2, 3, 6; 2, 2, 3, 5 1, 3, 3, 5; 2, 3, 3, 4 2, 2, 2, 6; 3, 3, 3, 3	1 point	*The condition about the mode forbids the following cases:* 1, 1, 4, 6; 2, 2, 3, 5 1, 2, 3, 6; 2, 2, 4, 4 1, 3, 3, 5; 2, 3, 3, 4 1, 3, 4, 4; 3, 3, 3, 3 2, 2, 2, 6;
Of the five remaining cases, the condition about the mode forbids 1, 1, 4, 6 and 1, 3, 4, 4 as well as 2, 2, 4, 4.	1 point	*Both of the remaining two cases satisfy the condition about the median.*
The results of the first four rolls must have been 1, 1, 5, 5 or 1, 2, 4, 5 (in any particular order).	1 point	
Total:	**6 points**	

Note: Award 1 or 2 points, respectively, if the candidate gives one or both correct solutions without any reasoning.

6. a)		
$i = r\alpha$, i.e. $\alpha = \frac{i}{r}$ (radians).	1 point	$i = \frac{\alpha}{360°} \cdot 2r\pi$, *i.e.* $\alpha = \frac{180°i}{r\pi}$.
As $r = 2$ cm, and so $i = (10 - 2 \cdot 2 =)$ 6 cm, $\alpha = \frac{6}{2} = 3$ radians.	2 points	$\alpha \approx 171.9°$
The area of the sector is $A = \frac{ir}{2} = 6$ cm^2.	1 point	$A = \frac{r^2\pi}{360°} \cdot \alpha \approx 6$ cm^2
The radius of the base circle of the cone is $R = \frac{i}{2\pi} = \frac{3}{\pi} \approx 0.95$ cm.	1 point	
Total:	**5 points**	

Note: Deduce a total 1 point if the candidate's calculations are correct but units are not given or not everywhere given.

6. b) Solution 1		
(Let A be the area of the sector.) As $A = \frac{ir}{2}$ and $i = 10 - 2r$,	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$A = \frac{(10-2r)r}{2} = 5r - r^2$.	1 point	
$A(r) = -(r-2.5)^2 + 6.25 \ (0 < r < 5)$	2 points*	*The graph of function A is a parabolic arc that intersects the x-axis in the origin and in the point* (5; 0).
As the first term is not positive, $A(r) \le 6.25$.	1 point*	*The parabola is inverted, due to the negative quadratic coefficient.*
$A(r)$ reaches its maximum where $(r-2.5)^2 = 0$, that is $r = 2.5$ (which is an element of the domain).	1 point*	*A maximum is reached at* 2.5, *the arithmetic mean of the two zeroes.*
If $r = 2.5$ cm, then $i = 5$ cm, i.e. $i = 2r$,	1 point	
in which case $\alpha = 2$ radians is true.	1 point	
Total:	**8 points**	

Notes:

1. The first two points of the solution may also be given for the following reasoning:

If the central angle is α degrees, then $i=\frac{\alpha}{180}\cdot r\pi$, so $10=\frac{\alpha}{180}\cdot r\pi+2r$, from where $\alpha=180\cdot\frac{10-2r}{r\pi}$.	1 point	*If the central angle is α radian then* $i=r\alpha$, *so* $10=r\alpha+2r$, *from where* $\alpha=\frac{10-2r}{r}$.
$A=\frac{\alpha}{360}\cdot r^2\pi=\frac{10-2r}{2r\pi}\cdot r^2\pi=5r-r^2$	1 point	$A=\frac{\alpha r^2}{2}=\frac{(10-2r)r^2}{2r}=$ $=5r-r^2$

*2. The points marked by * may also be given for the following reasoning:*

The derivative of $A(r)=5r-r^2$ $(0<r<5)$ is $A'(r)=5-2r$ $(0<r<5)$.	1 point	
Function A has an extreme value where its derivative is zero: $5-2r=0$,	1 point	
$r=2.5$ (which is an element of the domain).	1 point	
In case $r<2.5$, $A'(r)$ is positive, while it is negative where $r>2.5$. At 2.5 the function A has a maximum (both local and global).	1 point	$A''(r)=-2<0$ for the entire domain, so the function A has an (absolute) maximum at 2.5.

*3. The points marked by * may also be given for the following reasoning:*

The quadratic function $x\mapsto ax^2+bx+c$ $(a<0,$ $x\in\mathbf{R})$ assumes its maximum at $-\frac{b}{2a}$.	2 points	
The quadratic function $r\mapsto 5r-r^2$ $(r\in\mathbf{R})$ assumes its maximum at $-\frac{5}{-2}=2.5$.	1 point	
(2.5 is within the domain of the function A and so) $r=2.5$ is a maximum of A.	1 point	

4. Award a maximum of 7 points if the candidate works with rounded values.

6. b) Solution 2		
(Let A be the area of the sector.) If the central angle of the arc is 2 radian then $i = 2r$ and so (as $i = 10 - 2r$) $i = 5$ cm, $r = 2.5$ cm and $A = 6.25$ cm^2.	2 points	
It will be proven that in case the central angle of the arc is not 2 radian ($r \neq 2.5$) then the area of the sector is less than 6.25 cm^2.	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
$A = \frac{(10-2r)r}{2} = 5r - r^2$	1 point	
Consider the inequality $5r - r^2 < 6.25$ $(0 < r < 5)$.	1 point	
$0 < r^2 - 5r + 6.25$ $0 < (r - 2.5)^2$	1 point	
If $r \neq 2.5$ then the right side is positive and so the inequality holds true.	1 point	
The area of the sector will be maximal if the central angle is 2 radian.	1 point	
Total:	**8 points**	

6. c)		
The statement is false.	1 point	
Correct reasoning. (*A particular counterexample:* If, for example, $r_{10} = 3$ cm, $i_{10} = 4$ cm, then $A_{10} = 6$ cm^2. Let $r_{20} = 9.5$ cm, $i_{20} = 1$ cm, $A_{20} = 4.75$ cm^2. In this case $A_{10} > A_{20}$. *The analytic approach:* Given is an arbitrary sector of perimeter 10 cm. If ε is an arbitrarily small, positive number and the radius of the sector with the 20 cm perimeter is $r_{20} = 10 - \varepsilon$ then $i_{20} = 2\varepsilon$ and $A_{20} = (10 - \varepsilon)\varepsilon = 10\varepsilon - \varepsilon^2$. This could again be an arbitrarily small number also including being smaller than the area of the sector of perimeter 10 cm. *The practical approach:* Given is an arbitrary sector of perimeter 10 cm. The radius of the perimeter 20 cm sector may be “arbitrarily close” to 10 cm and so the length of the arc, and the area of the sector, too, may be an arbitrarily small positive number, perhaps even smaller than the area of the sector of perimeter 10 cm.)	2 points	
Total:	**3 points**	

7. a)		
The probability of selecting a green marble is $\frac{3}{4+3+s}$ (for both selections).	1 point	
$\left(\frac{3}{7+s}\right)^2 = 0.09$	1 point	
$\frac{3}{7+s} = 0.3$ (as $\frac{3}{7+s} > 0$),	1 point	
The number of yellow marbles is $s = 3$.	1 point	
Total:	**4 points**	

7. b) Solution 1		
There are $\binom{k+7}{3}$ different ways to select 3 out of $k + 7$ marbles (this is the total number of cases).	1 point	
There are 4 different ways to select a red marble, 3 to select a green, a k to select a blue one (which are all independent of one another), therefore there are $4\cdot3\cdot k = 12k$ different ways to select 3 marbles of different colours (the number of favourable cases).	2 points	
The probability is $\frac{12k}{\binom{k+7}{3}} =$	1 point	
$= \frac{12k}{\frac{(k+7)(k+6)(k+5)}{3\cdot2\cdot1}} = \frac{72k}{(k+7)(k+6)(k+5)}$	1 point	
Total:	**5 points**	

7. b) Solution 2		
The probability of selecting, for example, a red, a green and then a blue marble, in this particular order, is $\frac{4}{k+7}\cdot\frac{3}{k+6}\cdot\frac{k}{k+5}$.	1 point	
The different colours may also be selected in any other order. This does not change the denominators of the fractions, while their numerators will still be 4, 3 and k in one or the other order (their product being the same).	1 point	*This point is also due if the correct reasoning is reflected only by the solution.*
Three different colours may be selected in 3! different orders.	1 point	
The probability is $3!\cdot\frac{4}{k+7}\cdot\frac{3}{k+6}\cdot\frac{k}{k+5} =$	1 point	
$= \frac{72k}{(k+7)(k+6)(k+5)}$.	1 point	
Total:	**5 points**	

7. b) Solution 3		
(If the marbles are considered different) the total number of simple events (of equal probability) is $(k+7)(k+6)(k+5)$.	1 point	
A red, a green and a blue marble, in a certain particular order, may be selected in $4\cdot 3\cdot k\cdot 3!$ different ways (the number of favourable simple events).	2 points	
The probability is $\dfrac{4\cdot 3\cdot k\cdot 3!}{(k+7)(k+6)(k+5)}=$	1 point	
$=\dfrac{72k}{(k+7)(k+6)(k+5)}$.	1 point	
Total:	**5 points**	

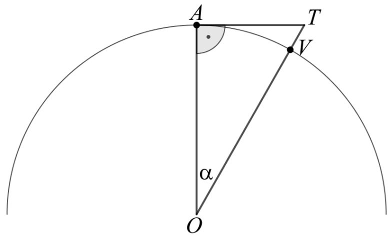
7. c)		
(According to the text $k\geq 3$. There are $\binom{k}{3}$ ways to select 3 out of k blue marbles, and there is a total of $\binom{k+7}{3}$ different selections altogether.) The probability of selecting three blue marbles is $\dfrac{\binom{k}{3}}{\binom{k+7}{3}}$.	2 points*	
As per the condition $\dfrac{\binom{k}{3}}{\binom{k+7}{3}}=\dfrac{72k}{(k+7)(k+6)(k+5)}$.	1 point*	$\dfrac{\binom{k}{3}}{\binom{k+7}{3}}=\dfrac{12k}{\binom{k+7}{3}}$
$\dfrac{\frac{k(k-1)(k-2)}{3\cdot 2\cdot 1}}{\frac{(k+7)(k+6)(k+5)}{3\cdot 2\cdot 1}}=\dfrac{72k}{(k+7)(k+6)(k+5)}$	1 point*	$\dfrac{k(k-1)(k-2)}{3\cdot 2\cdot 1}=12k$
This simplifies to $(k-1)(k-2)=72$ (as $k\neq 0$).	1 point	
One root of the equation $k^2-3k-70=0$ is negative (-7),	1 point	
the other root is the actual solution, $k=10$.	1 point	
Total:	**7 points**	

*Note: The 4 points marked with * may also be given for the following reasoning:*

(According to the text $k \geq 3$. The probability of selecting a blue marble first is $\frac{k}{k+7}$, the probability of selecting blue for second is $\frac{k-1}{k+6}$, for third it is $\frac{k-2}{k+5}$.) The probability of selecting three blue marbles is (the product of the above) $\frac{k}{k+7}\cdot\frac{k-1}{k+6}\cdot\frac{k-2}{k+5}$.	2 points	
As per the condition $\frac{k}{k+7}\cdot\frac{k-1}{k+6}\cdot\frac{k-2}{k+5}=\frac{72k}{(k+7)(k+6)(k+5)}$.	1 point	
$k(k-1)(k-2)=72k$	1 point	

8. a)		
Let h be the average depth of Lake Balaton. Calculating in meters: $2\cdot10^9=76.5\cdot10^3\cdot7.7\cdot10^3\cdot h$,	1 point	
$h=\frac{2\cdot10^9}{76.5\cdot10^3\cdot7.7\cdot10^3}$.	1 point	
Rounded correctly $h\approx3.4$ metres.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**3 points**	

8. b)		
Not counting lunch break, the cyclists rode for a total 11.5 hours. Let t be the time spent riding before the lunch break (in hours). In this case the time spent riding after lunch is $11.5 - t$.	1 point	
The boys rode $16t$ kilometres before lunch, $20\cdot(11.5 - t)$ kilometres after lunch.	1 point	
The total distance rode is $16t + 20\cdot(11.5 - t) = 205$,	1 point	
so $t = 6.25$ (hours).	1 point	
$7+6.25=13.25$; lunch lasted from quarter past one to quarter past two.	1 point	
Check: from seven o'clock to quarter past one they rode $16\cdot6.26 = 100$ km, from quarter past two to half past seven $20\cdot5.25 = 105$ km, a total $100 + 105 = 205$ km.	1 point	
Total:	**6 points**	

8. c)		
The plane across the centre of the Earth (O), Balatonalmádi (A) and Balatonvilágos (V) determines a great circle of Earth.	1 point	*This point is also due for a correct diagram.*
 In this diagram α is the central angle that belongs to arc AV of sector AOV, while segment VT is the signpost. The light is just visible in Balatonalmádi if the line TA is tangent to the circle, i.e. $OAT\angle = 90°$.	1 point	
Let i be the length of the minor arc AV. If R is the radius of the Earth then $\frac{i}{2R\pi} = \frac{\alpha}{360°}$,	1 point*	
$\alpha \left(= \frac{180° \cdot i}{R\pi} = \frac{180° \cdot 12.7}{6370 \cdot \pi} \right) \approx 0.114°$.	1 point*	
In the right triangle OAT $OT = \frac{OA}{\cos\alpha} \approx 6370.013$ (km).	1 point*	
The (minimal) height of the signpost above the water level is $VT \approx 6370.013 - 6370$ (km)	1 point	
that is about 13 metres.	1 point	*about* 12.7 *metres*
Total:	**7 points**	

Notes:
1. Deduce a total 1 point only if the candidate calculates with $\alpha = 0.11°$ *or* $\alpha = 0.1°$ *and gives the height of the signpost as approx.* 12 m (11.7 m) *or* 10 m (9.7 m).

*2. The points marked by * may also be given for the following reasoning:*

The lengths of the (minor) arc AV and the segment AT are approximately equal (12.7 km) as arc AV is “negligibly small” compared to the great circle of Earth (the central angle α of the arc AOV is very small).	2 points	*The length of segment AT (treating the data as exact values) is approximately* 12.70002 *km, that is, it is about 2 cm longer than the* 12.7 *km arc AT.*
Apply the Pythagorean Theorem in triangle OAT: $OT = \sqrt{6370^2 + 12.7^2} \approx 6370.013$ (km).	1 point	

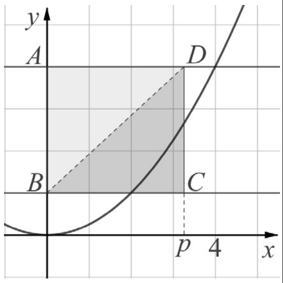
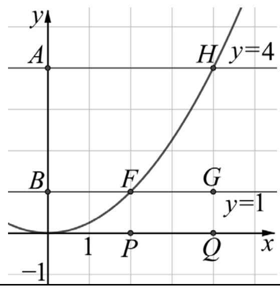
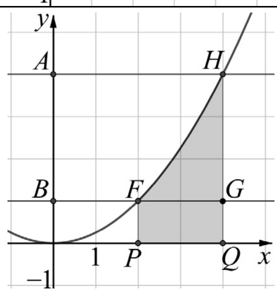
9. a)		
C is true	1 point	
B, D, E, F are false	2 points*	
Total:	**3 points**	

*Note: Deduce 1 point from the 2 points marked by * for each incorrect answer (but no more than a total of 2 points).*

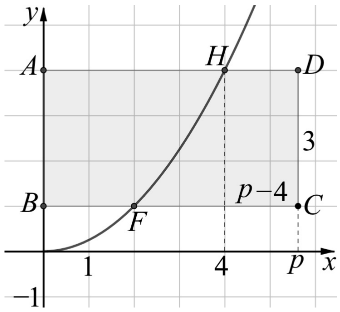
9. b) Solution 1		
Function a is strictly decreasing over the interval $]x_1; 0[$ and so the derivative function must be negative here. However, function b is positive over the whole interval $]x_1; 0[$,	2 points	
therefore it cannot be the derivative function of a. Statement **A** is false.	1 point	
Total:	**3 points**	

9. b) Solution 2		
In one point of the interval $]0; x_2[$ c has a zero and it changes sign here. According to statement **A**, function b would have to have a local extreme here. This is not true,	2 points	
so function c cannot be the derivative function of b. Statement **A** is false.	1 point	
Total:	**3 points**	

9. b) Solution 3		
Function a is convex (from above) over the whole domain and so its second derivative should be positive everywhere over the same interval. Function c is not everywhere positive	2 points	
therefore it cannot be the second derivative function of a. Statement **A** is false.	1 point	
Total:	**3 points**	

9. c)		
The parabola $y=\frac{x^2}{4}$ intersects the line $y = 4$ at $x = 4$ (also at $x = -4$). It intersects the line $y = 1$ at $x = 2$ (also at $x = -2$).	1 point	
If $p \le 4$, then the parabola may only split the right triangle BCD (“half” of the rectangle), and so the area that is being cut away by the parabolic arc must be smaller than half of the area of the rectangle, therefore $p > 4$. 	2 points	
Mark the points $F(2; 1)$, $G(4; 1)$ and $H(4; 4)$. Also mark $P(2; 0)$ and $Q(4; 0)$. 	1 point	*This point is also due for a correct diagram.*
The area below the parabolic arc over the interval [2; 4] is $A_P=\int_2^4 \frac{x^2}{4}dx=\left[\frac{x^3}{12}\right]_2^4=$ 	1 point	
$=\frac{64-8}{12}=\frac{14}{3}$,	1 point	
The area above the line $y = 1$ is $A_1=\frac{14}{3}-2\cdot 1=\frac{8}{3}$ (this is the difference between A_P and the area of the rectangle $FPQG$).	1 point	
The area of the figure bounded by the parabolic arc and the segments HA, AB, BF is: $A_2=A_{ABGH}-A_1=4\cdot 3-\frac{8}{3}=\frac{28}{3}$.	1 point*	
The area of rectangle $ABCD$ is twice as much: $A_{ABCD}=\frac{56}{3}\,(=AB\cdot p)$,	1 point*	
so $p=\frac{A_{ABCD}}{AB}=\frac{\frac{56}{3}}{3}=\frac{56}{9}$.	1 point*	
Total:	**10 points**	

*The points marked by * may also be given for the following reasoning:*

The area of the figure bounded by the parabolic arc and the segments *FC, CD, DH* is $\frac{8}{3}+3(p-4)$. 	1 point	
The area of rectangle *ABCD* is twice as much: $\frac{16}{3}+6(p-4)=3p$.	1 point	
So $p=\frac{56}{9}$.	1 point	

Note: Award a maximum of 8 points if the candidate does not examine the case p ≤ 4*, but rather considers p* > 4 *to be true only in light of their results.*